Radiative Neutrino Mass Generation and Neutrinoless Double Beta Decay

Julio

Oklahoma State University

Workshop on Major DUSEL Physics Topics
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Based on:

Two-loop neutrino mass generation through leptoquarks

Outline

- Motivation
- Two-loop neutrino mass model
- Neutrinoless double beta decay
- Lepton flavor violation
- New CP violation in $B_s$ system
- Conclusions
## Review of Neutrino Data

<table>
<thead>
<tr>
<th>( \Delta m^2_{21} ) (eV(^2))</th>
<th>( 7.65^{+0.23}_{-0.20} \times 10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\Delta m^2_{31}</td>
</tr>
<tr>
<td>( \sin^2 \theta_{12} )</td>
<td>( 0.304^{+0.022}_{-0.016} )</td>
</tr>
<tr>
<td>( \sin^2 \theta_{23} )</td>
<td>( 0.50^{+0.07}_{-0.06} )</td>
</tr>
<tr>
<td>( \sin^2 \theta_{13} )</td>
<td>( \leq 0.031(2\sigma) ) \hspace{1cm} ( \leq 0.047(3\sigma) )</td>
</tr>
</tbody>
</table>

Schwetz, Tortola, Valle, 2008;
Mezzetto & Schwetz, 2010
Open questions:

- Why are neutrino masses much smaller than other leptons?
- Why is $\theta_{13}$ small?
- What is neutrino mass hierarchy?
- Is new physics behind all of these?
The Origin of Neutrino Mass

Seesaw mechanism

\[ L = f_v L N^c H + M_R N^c N^c + h.c. \]

- There should be heavy states in order to generate neutrino mass.
- The required scale is beyond the LHC reach.

\[ m_\nu \approx \frac{v^2}{M_R} \rightarrow M_R \approx 10^{14} \text{ GeV} \]
Lower Scale of New Physics?

- Large mixing in neutrino sector but unobservable charged lepton LFV, $\text{BR (}\mu \rightarrow e\gamma\text{)} < 10^{-11}$.
- SM + seesaw = $\text{BR (}\mu \rightarrow e\gamma\text{)} \sim 10^{-48}$.
- Lower scale needs extra suppression.
- Loop can induce natural suppression
Radiative Neutrino Mass Generation

Zee Model

\[
m_{\nu} = \begin{pmatrix}
0 & m_{e\mu} & m_{e\tau} \\
m_{e\mu} & 0 & m_{\mu\tau} \\
m_{e\tau} & m_{\mu\tau} & 0
\end{pmatrix}
\]

\[\theta_{12} \approx \pi / 4\]

Ruled out by neutrino data

Koide, 2002; Frampton et al., 2002; He, 2004

Zee-Babu Model

\[
(m_{\nu})_{\text{largest}} \approx \frac{f^2 g}{(16\pi^2)^2} \frac{m_{\tau}^2}{\Lambda}
\]

• One of neutrinos is nearly massless.
• Inducing TeV scale of new physics.
• Still compatible with current data.

Babu & Macesanu, 2003; Aristizabal Sierra & Hirsch, 2006; Nebot et al., 2008
**ΔL=2 Operators**

- Essential to generate Majorana neutrino mass.
- The smallness of neutrino mass can be understood based on dimensional ground without knowing the detail of theory.
- The scale of NP can be predicted.

\[
\begin{align*}
O_1 &= L^i L^j H^k H^l \tilde{d}_{ik} \tilde{d}_{jl} & \text{Seesaw} \\
O_2 &= L^i L^j L^k e^c H^l \tilde{d}_{ij} \tilde{d}_{kl} & \text{Zee Model} \\
O_3 &= \{L^i L^j Q^k d^c H^l \tilde{d}_{ij} \tilde{d}_{kl}, L^i L^j Q^k d^c H^l \tilde{d}_{ik} \tilde{d}_{jl}\} \\
O_4 &= \{L^i L^j Q_i u^c H^k \tilde{d}_{jk}, L^i L^j Q_k u^c H^k \tilde{d}_{ij}\} \\
O_5 &= L^i L^j Q^k d^c H^l H^m \tilde{H}_i \tilde{d}_{jl} \tilde{d}_{km} \\
O_6 &= L^i L^j Q_k u^c H^i H^k \tilde{H}_i \tilde{d}_{ji} \\
O_7 &= L^i Q^j e^c \tilde{Q}_k H^k H^l H^m \tilde{d}_{il} \tilde{d}_{jm} \\
O_8 &= L^i e^c d^c u^c H^j \tilde{d}_{ij} \\
O_9 &= L^i L^j L^k e^c L^l e^c \tilde{d}_{ij} \tilde{d}_{kl} & \text{Zee-Babu Model}
\end{align*}
\]

Babu & Leung, 2002
Neutrino Mass Model

- Needs two-loop to generate neutrino mass
  - \( m_\nu \sim \frac{m_t m_b m_\tau \nu}{(16\pi^2)^2 \Lambda^3} \)

- TeV scale of NP
- Needs leptoquarks
Model

- The operator $O_8$ can be generated by

$$\mathcal{L} = Y_{ij} L_i^\alpha d_j^c \Omega^\beta \bar{\phi}_{\alpha\beta} + F_{ij} e_i^c u_j^c \chi^{-1/3} + \mu \Omega^\dagger H \chi^{-1/3} + \text{h.c.}$$

- Introducing two leptoquarks

$$\Omega \equiv \begin{pmatrix} \omega^{2/3} \\ \omega^{-1/3} \end{pmatrix} \sim (3, 2, 1/3), \quad \chi^{-1/3} \sim (3, 1, -2/3)$$

$$\begin{pmatrix} \omega^{-1/3} \\ \chi^{-1/3} \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} X_1^{-1/3} \\ X_2^{-1/3} \end{pmatrix} \Rightarrow \sin 2\theta = \frac{2\mu v}{M^2_2 - M^2_1}$$

$$M^2_{1,2} = \frac{1}{2} \left[ m_\omega^2 + m_\chi^2 \pm \sqrt{(m_\omega^2 - m_\chi^2)^2 + 4\mu^2 v^2} \right]$$
Neutrino Mass Model

- The Goldstone mediated diagrams cancel due to the relative minus sign in $u-d$ vertex.
- The neutrino mass

$$(M_{\nu})_{ij} = \hat{m}_0 \left[ Y_{ik} (D_d)_k (V^T)_{kl} (D_u)_l (F^\dagger)_{ij} (D_\ell)_{j} + \text{transpose} \right] I_{jkl}$$

$$\hat{m}_0 = \left( \frac{Cg^2 \sin 2\theta}{(16\pi^2)^2} \right) \left( \frac{m_1 m_b m_\tau}{M_1^2} \right); \quad C = 3$$
$I_{jkl}$ is the dimensionless loop integral

$$I_{jkl} = \frac{M_1^2}{m_W^2 - m_{\nu_j}^2} \sum_{a=1,2} (-1)^a \int_0^1 dx \int_0^\infty t dt \left( \frac{1}{t + M_a^2} - \frac{1}{t + m_k^2} \right) \times \ln \left[ \frac{m_W^2 (1 - x) + m_{\nu_j}^2 x + tx(1 - x)}{m_{\nu_j}^2 (1 - x) + m_{\nu_j}^2 x + tx(1 - x)} \right]$$

![Graph showing the relationship between $I(M_1, M_2)$ and $M_1$ (TeV)]
Neutrino Mass Matrix

- Neutrino mass matrix has normal hierarchy texture.
- The (1,1) element is suppressed, i.e. \( \ll 0.1 \text{ eV} \).
- For most of parameter range, \( w \ll 1 \).
- For \( w \ll 1 \), the \( \det (M_\nu) \approx 0 \).
- \( w \) may be significant for LQ mass less than 500 GeV.

\[
(M_\nu)_{11} = y \left( \frac{F_{13}^*}{F_{33}^*} \right) \left( \frac{m_e}{m_\tau} \right) m_0
\]

\[
M_\nu = m_0 \begin{pmatrix}
0 & \frac{1}{2} m_\mu \cdot \frac{x}{m_\tau} & \frac{1}{2} y \\
\frac{1}{2} m_\mu \cdot \frac{x}{m_\tau} & \frac{m_\mu}{m_\tau} \cdot x z & \frac{1}{2} z + \frac{1}{2} \frac{m_\mu}{m_\tau} x \\
\frac{1}{2} y & \frac{1}{2} z + \frac{1}{2} \frac{m_\mu}{m_\tau} x & 1 + w
\end{pmatrix}
\]

\[
x = \frac{F_{23}^*}{F_{33}^*}, \quad y = \frac{Y_{13}}{Y_{33}}, \quad z = \frac{Y_{23}}{Y_{33}}
\]

\[
w = \frac{F_{32}^*}{F_{33}^*} \frac{Y_{32}}{Y_{33}} \left( \frac{m_c}{m_t} \right) \left( \frac{m_s}{m_b} \right) \frac{I_{jkl}}{I_{jk3}}
\]

\[
m_0 = 2 \hat{m}_0 F_{33}^* Y_{33} I_{jk3}
\]
The (nearly) vanishing determinant indicates \( m_1 \) is (nearly) massless.

- The \( \theta_{13} \) is predicted to be large.

- This prediction can be tested by long baseline experiments.
Measuring $\delta$

- The interesting LQ decay modes:

$$\Gamma(\omega^{2/3} \rightarrow e^+ b) : \Gamma(\omega^{2/3} \rightarrow \mu^+ b) : \Gamma(\omega^{2/3} \rightarrow \tau^+ b) = |y|^2 : |z|^2 : 1$$

$$\Gamma(X_a^{-1/3} \rightarrow \mu^- t) : \Gamma(X_a^{-1/3} \rightarrow \tau^- t) = |x|^2 : 1$$

- Provide way to measure $\delta$.
- Neutrino fit predicts that $\omega^{2/3}$ decay into lepton and b-quark are the same for all flavors.
- LQ $X^{-1/3}$ prefers to decay into muon than tau.
A special case for $w \gg 1$

- This is restricted to small leptoquark mass, i.e. $M_{LQ} \leq 500 \text{ GeV}$.

- Lead to the known texture with $M_{11} = 0$ and $M_{13} = 0$.
  Frampton et al., 2002

- The $\theta_{13}$ is shifted down.

\[ 0.012 \leq \sin^2 \theta_{13} \leq 0.04 \]

- Can be cross-checked by long baseline experiments.
Neutrinoless Double Beta Decay

The vanishing of (1,1) element of neutrino mass matrix causes the $0\nu\beta\beta$ cannot occur through “standard mechanism”

\[ M \propto (M_\nu)_{11} \]
Neutrinoless Double Beta Decay

- Although $M_{11} \approx 0$, neutrinoless double beta decay may still occur through “vector-scalar exchange”.

\[
\mathcal{L}_{\text{eff}} = \frac{G_F^2}{2} \dot{\bar{u}} \gamma^\mu (1 - \gamma_5) d \left[ \bar{u}(1 + \gamma_5) d e^\mu (1 - \gamma_5) \frac{1}{4} \sigma^{\alpha\beta} e^\alpha + \frac{1}{4} \bar{u} \sigma^{\alpha\beta} (1 + \gamma_5) d e^\mu (1 - \gamma_5) \frac{1}{4} \sigma^{\alpha\beta} e^\alpha \right]
\]

\[
\dot{\mathcal{O}} = \frac{y_{11}^* F_{11}}{2\sqrt{2} M_1^2 G_F} \sin 2\theta \left( 1 - \frac{M_1^2}{M_2^2} \right)
\]

\[
|y_{11}^* F_{11}| < 1.7 \times 10^{-6} \left( \frac{M_1}{1 \text{ TeV}} \right)^2 \left( \frac{0.5 \text{ TeV}}{\mu} \right)
\]

- $0\nu\beta\beta$ could occur although this model features normal hierarchy

Babu & Mohapatra, 1995
Paes et al., 1996
**μ-e Conversion in the Nuclei**

<table>
<thead>
<tr>
<th>Element</th>
<th>BR</th>
<th>Constraint</th>
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<tbody>
<tr>
<td>$^{48}$Ti</td>
<td>$&lt; 4.3 \times 10^{-12}$</td>
<td>$\left</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>$&lt; 4.6 \times 10^{-11}$</td>
<td>$\left</td>
</tr>
</tbody>
</table>

For $^{48}$Ti at 1 TeV

$|Y_{13}^* Y_{23}| < 10^{-3}$  $|F_{13} F_{23}^*| < 10^{-4}$

$|Y_{11}^* Y_{21}| < 10^{-6}$  $|F_{11} F_{21}^*| < 10^{-6}$  

Tree level

The COMET and Project-X may see this process.
None of the couplings is allowed to exceed one.

- LQ mass cannot exceed $10 \text{ TeV}$ or else the induced neutrino mass is too small.

\[ | Y_{13}^* Y_{23} | < 10^{-3} \quad | F_{13}^* F_{23}^* | < 10^{-4} \]

- $Y_{13} \sim Y_{23} \sim Y_{33}$ while $F_{23} \sim 16F_{33}$ from neutrino mass fitting.
- None of the couplings is allowed to exceed one.
- LQ mass cannot exceed 10 TeV or else the induced neutrino mass is too small.
$\mu \rightarrow e \gamma$

- For $\omega^{2/3}$ mediated process, there is GIM-like cancellation.
- No lower limit for $F_{13}$, so the smallest branching ratio may be zero.
- This process may not be seen by MEG experiment.

$$\frac{|\xi_1(x_b)Y_{13}^*Y_{23}|^2}{m^4_\omega} + \frac{|\frac{1}{12}F_{11}^*F_{21} + \frac{1}{12}F_{12}^*F_{22} + \xi_2(x_f)F_{13}^*F_{23}|^2}{m^4_\chi} < \frac{3.1 \times 10^{-19}}{\text{GeV}^4}$$

$$\xi_1(x) = \frac{x(1-x)(5+x) + 2(2x+1)\ln x}{12(1-x)^4}$$

$$\xi_2(x) = \frac{1}{12} \frac{(1-x)(5x+1) + 2x(2+x)\ln x}{(1-x)^4}$$

$$x_f = \frac{m^2_f}{m^2_{LQ}}$$
New CP Violation in $B_s$ System

$$(A_{sl}^b)^{\text{obs}} = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3}$$

$$(A_{sl}^b)^{\text{SM}} = -2.3^{+0.5}_{-0.6} \times 10^{-4}$$

- New D0 observation in dimuon asymmetry indicates new physics affecting $B_{s,d}$ mixing.

$$\Delta M_{s,d} = \Delta M_{s,d}^{\text{SM}} + \Delta M_{s,d}^{\text{new}} = \Delta M_{s,d}^{\text{SM}} \left| 1 + h_{s,d} e^{2i\sigma_s} \right|$$

- The best fit shows only $B_s$ is affected.

$$\{h_s = 0.5, \sigma_s = 120^\circ\} \quad \{h_s = 1.8, \sigma_s = 100^\circ\}$$

(Ligeti et al., 2010)
New CP Violation in $B_s$ System

- This model could induce $B_s$ mixing,

\[ h_s e^{2i\sigma_s} = \frac{(Y_{i2}Y_{i3}^*)^2}{384\pi^2 m_\omega^2 M_{12s}^{SM}} m_B f_{B_s}^2 B_{1}^{B_1} (\mu) \eta_1^{B_s} (\mu) \]

\[ Y_{33} \sim 0.078, \quad Y_{32} \sim 1, \quad m_\omega = 300 \text{ GeV} \quad \rightarrow \quad h_s \sim 0.42 \]

Phase is not constrained, so can take $120^\circ$

- Optimum with tau inside the loop and $m_\omega < 500$ GeV
Conclusions

- This model could naturally generate small neutrino mass at TeV scale.
- The neutrino mass spectrum is predicted to be hierarchical.
- For most of parameter range, $\theta_{13}$ is predicted to be near the upper limit.
- Neutrinoless double beta decay may occur although it has normal hierarchy texture.
- New CP violation in $B_s$ system can be explained for light leptoquark.